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Modelling opportunistic spectrum renting in mobile cellular networks

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ABSTRACT

Spectrum pooling enables the opportunistic usage of licensed frequency bands for secondary users during the idle periods of primary users, which can be applied to relieve temporary bottlenecks in mobile cellular networks. In this paper, we provide an analytical framework for the performance evaluation of the interaction of multiple operators who cooperate and apply an opportunistic spectrum renting scheme. Based on our investigation, we provide a flexible and simple cooperation scheme when more than two operators are involved.

1. Introduction

The 2002 report of the Federal Communications Commission's (FCC) Spectrum Policy Task Force (Spectrum Policy Task Force, 2002) recommended a spectrum policy reform that is based on the flexible management of spectrum and liberalized/dynamic spectrum access. The idea of dynamic spectrum access is a radical approach compared to the current centralized spectrum management, that the exclusive access right of certain frequency bands is guaranteed to a specific operator. Since the release of the FCC report (Spectrum Policy Task Force, 2002), a quest for dynamic spectrum access technique to improve a spectrum efficiency has been intensively researched (see Spectrum Policy Task Force, 2002; Jabbari et al., 2010; Gandhi et al., 2008; Buddhikot, 2007; Zhao and Sadler, 2007; Peha, 2009; Weiss and Jondral, 2004; Talay and Altilar, 2013; Parvin et al., 2012 and references therein). Buddhikot (2007), Zhao and Sadler (2007), and Peha (2009) provided overviews and rationales concerning efficient spectrum sharing and access.

Although it is widely recognized that a spectrum management reform and dynamic spectrum access can provide a solution to the existing problem (i.e., the shortage of usable radio frequencies and the under-utilization of the licensed spectrum) (Spectrum Policy Task Force, 2002; Buddhikot, 2007; Zhao and Sadler, 2007), the application of dynamic and opportunistic spectrum access is rarely found in practice. A lot of issues (Spectrum Policy Task Force, 2002; Buddhikot, 2007; Zhao and Sadler, 2007; Weiss and Jondral, 2004) must be solved before the widespread application of opportunistic spectrum access.

Spectrum pooling is first proposed by Mitola (1999), which enables the secondary usage of the licensed frequency bands. The basic idea of the spectrum pooling is that secondary users may opportunistically utilize licensed frequency bands of primary users during the idle periods of primary users. Weiss and Jondral (2004) proposed a practical approach that allows the access of already licensed frequency bands without requiring any change to the licensed systems. The authors (Weiss and Jondral, 2004) described their solution for frequency-/time-division multiple access (FDMA/TDMA) based licensed systems. Furthermore, they (Weiss and Jondral, 2004) argued that spectrum pooling does not create a competition to existing and upcoming 2G and 3G mobile radio standards because a capacity shortage may happen due to a high demand in an area with limited frequency bands. In such cases, a specific operator can apply the spectrum pooling technique to enhance the grade of service for calls (i.e., to rent a free frequency band from another operator to serve incoming calls) (Tzeng and Huang, 2010; Tzeng, 2009; Do et al., 2012).

Some queueing models for spectrum renting were worked out (see Tzeng and Huang, 2010; Tzeng, 2009; Do et al., 2012). Tzeng (2009) and Tzeng and Huang (2010) assumed that user channels can be rented in one unit, which is not realistic because the separate blocks of user channels are defined in each frequency band and each block should be controlled by a single network operator. Do et al. (2012) presented the first queueing model to take into account this

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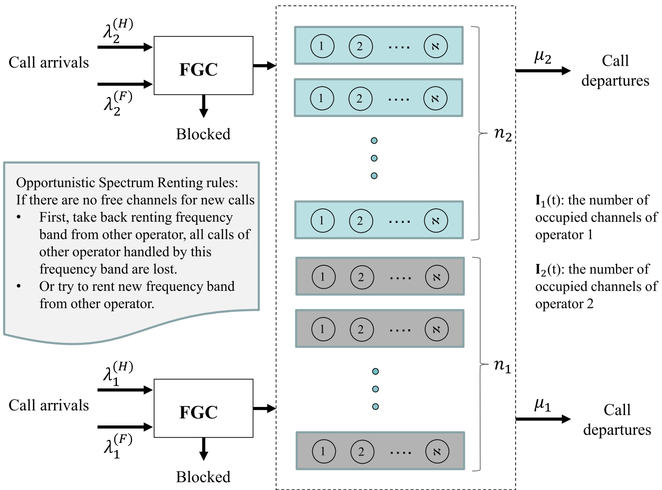

Fig. 1. Resource contention and spectrum renting in the case of two operators.

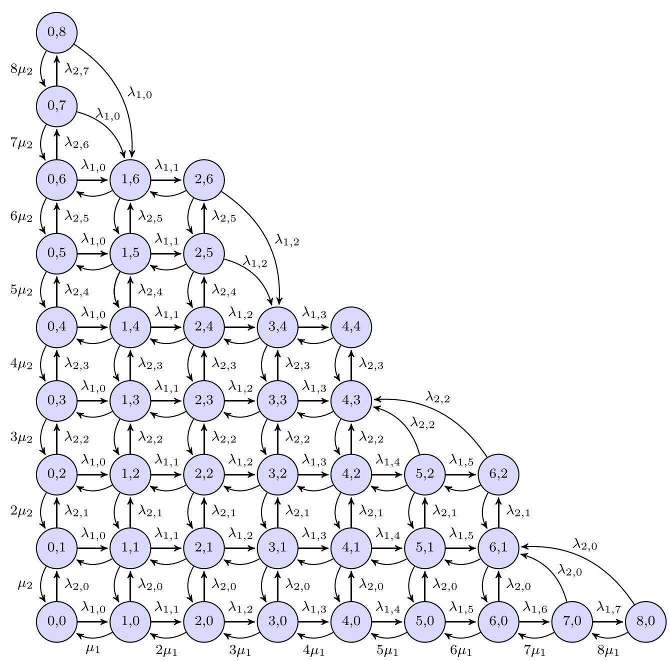

Fig. 2. Illustration of the state transition diagram.

technology aspect. Do et al. (2014) analyzed the retrial phenomenon with spectrum renting. However, they (Do et al., 2012, 2014) did not consider directly the interaction of operators.

In this paper, we provide an analytical framework for the performance evaluation of the interaction of multiple operators who cooperate and apply opportunistic spectrum renting in a specific area. For the ease of comprehension, we investigate a scenario where two operators apply an opportunistic spectrum pooling scheme in a specific area. We show that the proposed queueing model can accurately evaluate the performance of mobile cellular networks with call durations following the lognormal distribution. Based on our investigation, we provide a flexible and simple cooperation scheme when more than two operators are involved. The proposed scheme along with its simplicity can provide a good opportunity for operators to enhance a grade of service provisioned to subscribers, which could give an incentive for operators to cooperate together. To our best knowledge, this paper is the first work on modelling the interaction of multiple operators.

The rest of this paper is organized as follows. An overview on technical problems regarding spectrum pooling is provided in Section 2. A performance evaluation framework is presented in Section 3. The performance analysis and a numerical study in a case with two operators are presented in Sections 4 and 5, respectively. A general case when more than two operators are involved is discussed in Section 6. Finally, Section 7 concludes the paper.

2. A background on spectrum pooling

To realize a spectrum pooling system, several technical problems should be handled (Weiss and Jondral, 2004):

- First, the cooperating parties should be able to identify the idle spectral ranges for the secondary usage of the spectrum. To achieve this, a detection algorithm was proposed in Weiss et al. (2003a). Moreover, the diversity approach must be applied (Weiss et al., 2003a) to achieve the desired detection, which requires that all associated mobile terminals should perform spectral measurements.
- Second, the previously mentioned diversity approach involves an enormous signaling overhead which makes the system error-prone as interference will be occurred with the new licensed users. In Weiss et al. (2003b), a boosting protocol is proposed, which moves this signaling from the MAC layer to the physical layer.
- The third obstacle is the well known mutual interference problem. A possible solution of the mutual interference problem is the introduction of adaptive guard bands in the rental system. However, this method reduces the available bandwidth of the rental system. By all means, a trade-off may be found between the interference and the bandwidth of the rental system (Weiss and Jondral, 2004). In Weiss et al. (2003c), the authors propose the adaptive filtering of the narrowband interferers.

Furthermore, the investment (on the licensed frequencies and technology) protection of the incumbent operators plays an important role concerning the acceptance of opportunistic spectrum access. Weiss and Jondral (2004) proposed a spectrum pooling scenario where secondary users may temporarily rent frequency bands (a pool of bands) owned by licensed users during the idle periods of licensed users. In this way, spectrum pooling can be used to enhance the utilization of spectrum without any change to the licensed systems, which gives a good incentive for the acceptance of the spectrum pooling idea by the incumbent operators. The challenges, the technical requirements and possible solutions for the realization (including frequency-/time-division multiple access, the spectral shape of the transmitted signal, the detection of spectral access, collection and broadcast of spectral measurements, synchronization, etc.) of opportunistic spectrum access are discussed in Weiss and Jondral (2004).

3. A performance evaluation framework

Assume that there are K mobile cellular network operators in a certain area. Let n_k be the number of licensed frequency bands of operator k, $(k=1,\ldots,K)$, in the area. The number of channels is $\aleph$ in each frequency band. Therefore, the number of channels owned by operator k is $N_k = n_k\aleph$ for $k=1,\ldots,K$. The total number of channels in a specific area is $N=\sum_{k=1}^{K} N_k$.

To construct a mathematically tractable model within the Markovian framework, we follow the classical approach frequently applied in the queueing theory for the performance evaluation of wireless cellular networks (Tzeng and Huang, 2010; Tzeng, 2009;

Tran-Gia and Mandjes, 1997; Artalejo and Lopez-Herrero, 2010; Do, 2010, 2011). Fresh calls and handover calls arrive according to Poisson processes. The inter-arrival times of new calls and handover calls are exponentially distributed with rates $\lambda_k^{(F)}$ and $\lambda_k^{(H)}$, $(k=1,\ldots,K)$, respectively. We assume that call durations (of new admitted calls and handover admitted calls) are exponentially distributed with mean $1/\mu_k$, $(k=1,\ldots,K)$.

Let $I_k(t)$ be the number of busy channels handled by operator k, $(k=1,\ldots,K)$, at time instant t. A fresh call that requires the service of operator k, $(k=1,\ldots,K)$, is admitted according to the Fractional Guard Channel (FGC) policy (Do et al., 2012) to ensure a protection for handover calls. Let $\beta_{k,l}$, $(k=1,\ldots,K)$, denote the probability that a fresh call (arriving at time instant t) is accepted by operator k if the number of busy channels handled by operator k is $I_k(t)=l$. Note that a handover call is always accepted under the Fractional Guard Channel (FGC) policy (Do et al., 2012) when there is a non-allocated channel (a handover call is blocked when all channels are busy and a new frequency could not be rented). Therefore, the arrival rate of calls to operator k, $(k=1,\ldots,K)$, is $\lambda_{k,l}=\beta_{k,l}\lambda_k^{(F)}+\lambda_k^{(H)}$ in a specific area, when the number of busy channels handled by operator k is $I_k(t)=l$ and there is a free channel. Denote $\lambda_k=\lambda_k^{(H)}+\lambda_k^{(F)}$.

We assume that operators follow an etiquette to minimize the number of occupied frequency bands. To reach a goal that $\lceil I_k(t)/\aleph \rceil$ is the number of frequency bands occupied by operator k, $(k=1,\ldots,K)$ at time t,

- a newly arriving call is not placed to an idle frequency band, if there is a free channel in an occupied band,
- a reallocation of ongoing calls using frequency hopping may be performed upon the departure of a certain call.

Based on the assumptions, the system is described by the Continuous Time Markov Chain (CTMC) $Y=\{I_1(t),I_2(t),\ldots,I_K(t)\}$ on the state space $S=\{(i_1,\ldots,i_K)\,|\,0\le i_1\le N,\ldots,0\le i_K\le N,\sum_{k=1}^{K}\lceil i_k/\aleph\rceil\le\sum_{k=1}^{K}n_k\}$. The steady state probabilities are denoted by $p_{i_1,\ldots,i_K}=\lim_{t\to\infty}\Pr(I_1(t)=i_1,\ldots,I_K(t)=i_K)$.

State transitions only take place if a call arrives or an ongoing call departs. For a case with two operators (see Fig. 1), each operator owns 2 frequency bands and each frequency band has 2 channels, the illustration of the state space and the transitions of the CTMC is depicted in Fig. 2. For $K>2$, we assume that each operator should specify a rule regarding a preferable tenant operator (e.g., in the form of a preference list or a tenant operator is randomly selected). Furthermore, a rule to claim back a frequency band is also needed when several frequency bands are rented by multiple operators. As a consequence the state transitions balance equations can be determined, then the steady state distribution and the performance measures can be computed with the help of standard techniques (Takacs, 1962; Bolch et al., 2006).

Alternatively, the state space can be organized into the lexicographic order $(S_0,\ldots,S_N)$, where S_{i_1}, $0\le i_1\le N$, is the

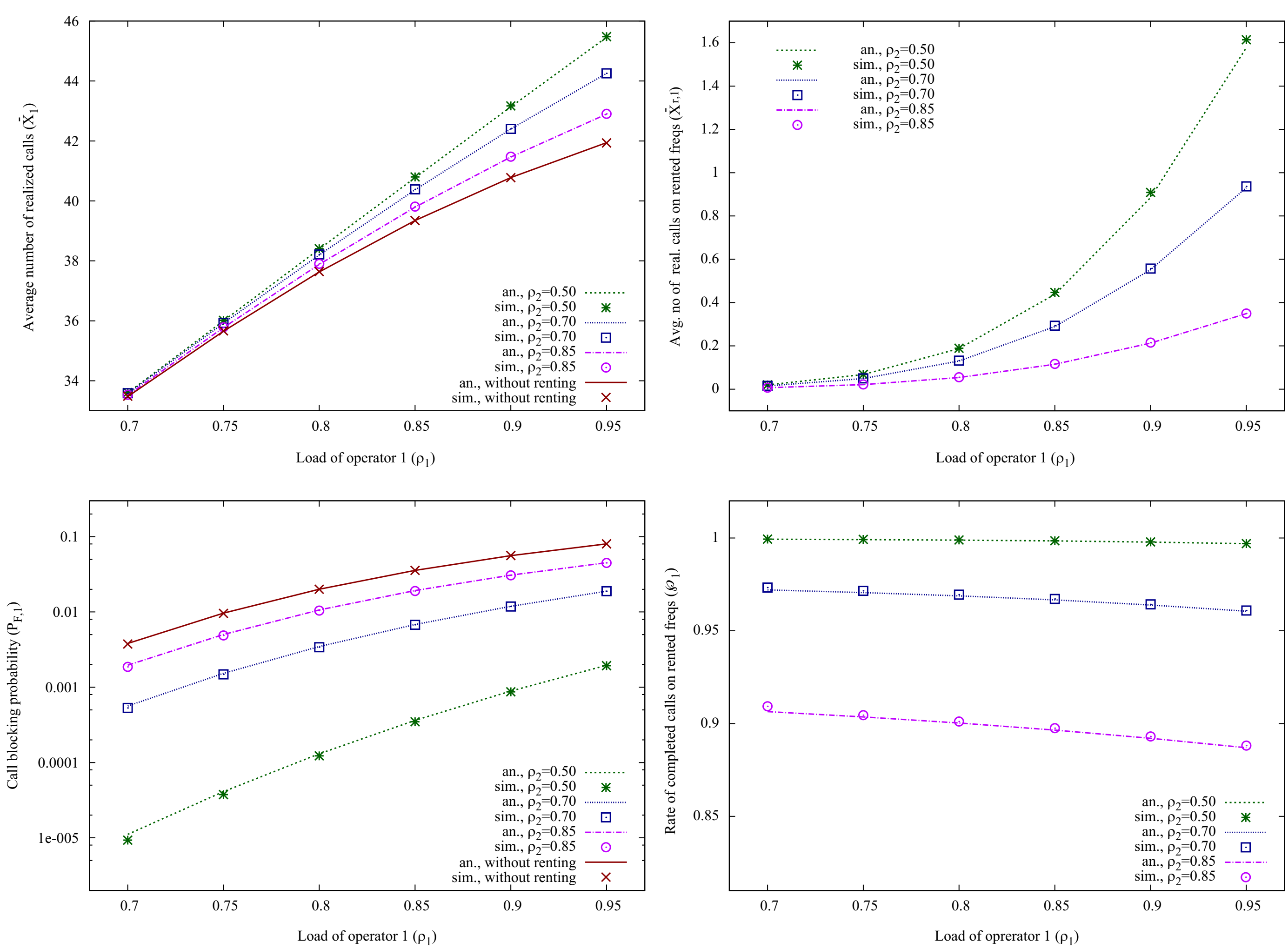

Fig. 3. Comparison with simulation ($n_1=n_2=6$, $1/\mu_1=1/\mu_2=53.22$ s).

set of states when $I_1(t)=i_1$. That is, $S_{i_1}=\{(i_1,0,\ldots,0),(i_1,0,\ldots,1),\ldots,(i_1,0,\ldots,N),(i_1,0,\ldots,1,0),\ldots(i_1,N,\ldots,N)\}$. CTMC Y can be interpreted as a two-dimensional Markov chain under the new lexicographic order. Therefore, the steady-state probabilities can be obtained by following the approach presented in Do et al. (2012). To ease the comprehension, we present the analysis for $k=2$ in Section 4 and for $k=3$ in the e-companion. Based on numerical results in Section 5, we provide a simple operation rule in Section 6.

4. A case with two operators ($K=2$)

We propose an opportunistic spectrum sharing scheme for two operators (see Fig. 1) in a specific area as follows. Both operators mutually act as licensed users and secondary users. Their licensed spectrum bands form a pool two operators can opportunistically use. It is worth emphasizing that the tenant operator can only occupy a frequency band owned by the renter if the specific band is idle. Furthermore, the tenant operator should vacate all calls from the rented band if the renter needs the specific band (i.e., to place ongoing calls to its own frequency band if it is possible). Calls that could not be reallocated are forced to terminate. Therefore, the tenant operator should increase the utilization of its own frequency bands as much as possible to reduce a chance that calls are forced to leave the system.

4.1. The stationary distribution

The following types of transitions are possible between the states of the CTMC (see Fig. 2):

- $(i_1,i_2)\Rightarrow(i_1+1,i_2)$ for $0\le i_1<N$, $0\le i_2<N$ and $\lceil (i_1+1)/\aleph\rceil+\lceil i_2/\aleph\rceil\le n_1+n_2$. These transitions are due to the acceptance of a call for the second provider,
- $(i_1,i_2)\Rightarrow(i_1,i_2+1)$ for $0\le i_2<N$, $0\le i_1<N$ and $\lceil i_1/\aleph\rceil+\lceil (i_2+1)/\aleph\rceil\le n_1+n_2$. These transitions are due to the acceptance of a call for the first provider,
- $(i_1,i_2)\Rightarrow(i_1-1,i_2)$ for $i_1>0$: these transitions happen when a call of the first provider departs from the system,
- $(i_1,i_2)\Rightarrow(i_1,i_2-1)$ for $i_2>0$: these transitions happen when a call of the second provider departs from the system,
- $(i_1,i_2)\Rightarrow(i_1+1,(\lceil i_2/\aleph\rceil-1)\aleph)$ for $\mathrm{mod}(i_1)=0$, $i_1<N_1$ and $\lceil i_1/\aleph\rceil+\lceil i_2/\aleph\rceil=n_1+n_2$, where $\mathrm{mod}(i_1)=i_1 \bmod \aleph$. These transitions are initiated by the arrival of a call and one rented frequency is taken back in case of the first provider,
- $(i_1,i_2)\Rightarrow((\lceil i_1/\aleph\rceil-1)\aleph,i_2+1)$ for $\mathrm{mod}(i_2)=0$ and $i_2<N_2$ and $\lceil i_1/\aleph\rceil+\lceil i_2/\aleph\rceil=n_1+n_2$. These transitions are initiated by the arrival of a call and one rented frequency is taken back in case of the second provider.

The matrix $(N+1-\lceil i_2/\aleph\rceil\aleph)\times(N+1-\lceil i_2/\aleph\rceil\aleph)$ A_{i_2} $(i_2=0,\ldots,N)$ contains the transition rate $A_{i_2}(i_1,i_1')$ from state (i_1,i_2) to state (i_1',i_2). We can write

$$A_{i_2}(i_1,i_1')=\begin{cases}\lambda_{1,i_1} & \text{if } i_1'=i_1+1,\ 0\le i_1<N,\\ i_1\mu_1 & \text{if } i_1'=i_1-1,\ 0<i_1\le N,\\ 0 & \text{otherwise.}\end{cases}$$

The $(N+1-\lceil i_2/\aleph\rceil\aleph)\times(N+1-\lceil (i_2+1)/\aleph\rceil\aleph)$ matrix B_{i_2} $(i_2=0,\ldots,N-1)$ includes the rate of transitions from state (i_1,i_2) to state (i_1',i_2+1). We obtain

$$B_{i_2}(i_1,i_1')=\begin{cases}\lambda_{2,i_2} & \text{if } i_1'=i_1,\\ \lambda_{2,i_2} & \text{if } \mathrm{mod}(i_2)=0 \text{ and } i_2<N_2,\\ & i_1'=(\lceil i_1/\aleph\rceil-1)\aleph,\\ & \lceil i_1/\aleph\rceil+\lceil i_2/n\rceil=n_1+n_2,\\ 0 & \text{otherwise.}\end{cases}$$

The elements of the $(N+1-\lceil i_2/\aleph\rceil\aleph)\times(N+1-\lceil (i_2-1)/\aleph\rceil\aleph)$ matrix C_{i_2} $(i_2=1,\ldots,N)$ are the rates of transitions from state (i_1,i_2) to state (i_1',i_2-1) due to the departure of a call. We obtain

$$C_{i_2}(i_1,i_1')=\begin{cases}i_2\mu_2 & \text{if } i_1'=i_1,\\ 0 & \text{otherwise.}\end{cases}$$

For $i_2>N_2$, the transition rate from state (i_1,i_2) to state $(i_1',(\lceil i_2/\aleph\rceil-1)\aleph)$ due to the arrival of a call to the first provider is included in matrix $C_{i_2,(\lceil i_2/\aleph\rceil-1)\aleph}=C_{i_2,*}$ of size $(N+1-\lceil i_2/\aleph\rceil\aleph)\times(N+1-(\lceil i_2/\aleph\rceil-1)\aleph)$ with only one nonzero element $C_{i_2,(\lceil i_2/\aleph\rceil-1)\aleph}(i_1,i_1+1)=\lambda_{1,i_1}$ if $\lceil i_1/\aleph\rceil+\lceil i_2/\aleph\rceil=n_1+n_2$ and $\mathrm{mod}(i_1)=0$.

Define

$$A_{i_2}^{(1)}=\begin{cases}A_0-D^{A_0}-D^{B_0} & \text{if } i_2=0,\\ A_{i_2}-D^{A_{i_2}}-D^{B_{i_2}}-D^{C_{i_2}} & \text{if } 0<i_2\le N_2,\\ A_{i_2}-D^{A_{i_2}}-D^{B_{i_2}}-D^{C_{i_2}}-D^{C_{i_2,*}} & \text{if } N_2<i_2<N,\\ A_N-D^{A_N}-D^{C_N}-D^{C_{N,*}} & \text{if } i_2=N.\end{cases}$$

Note that $D^Z(Z=A_{i_2},B_{i_2},C_{i_2})$ is a diagonal matrix whose diagonal element is the sum of all elements in the corresponding row of Z.

Let us introduce $\pi_{i_2}=(p_{0,i_2},\ldots,p_{f(i_2),i_2})$, where $f(i_2)=N-\lceil i_2/\aleph\rceil\aleph$. The balance equations, which equate the probability fluxes from and to the states of CTMC, can be written as follows:

$$\pi_0 A_0^{(1)}+\pi_1 C_1=0. \tag{1}$$

For $N_2\le i_2<N$ and $\mathrm{mod}(i_2)=0$, we have

$$\pi_{i_2-1}B_{i_2-1}+\pi_{i_2}A_{i_2}^{(1)}+\pi_{i_2+1}C_{i_2+1}+\sum_{l=1}^{\aleph}\pi_{i_2+l}C_{i_2+l,*}=0. \tag{2}$$

For $0<i_2<N$ and $\mathrm{mod}(i_2)\ne0$, we can write

$$\pi_{i_2-1}B_{i_2-1}+\pi_{i_2}A_{i_2}^{(1)}+\pi_{i_2+1}C_{i_2+1}=0. \tag{3}$$

The last balance equation is

$$\pi_{N-1}B_{N-1}+\pi_N A_N^{(1)}=0. \tag{4}$$

We can directly solve the balance equations and the normalization for the steady state probabilities. However, the computation complexity is $O((N+1)^6)$. To exploit the property of the matrices for the computation of the steady state probabilities, we shall proceed as follows. Let us define matrices R_{i_2}'s such as $\pi_{i_2}=\pi_{i_2-1}R_{i_2}$ for $i_2=1,2,\ldots,N$. Therefore, π_{i_2} $(i_2=1,2,\ldots,N)$ can be expressed in π_0 as

$$\pi_{i_2}=\pi_0\prod_{j=0}^{i_2}R_j,\quad (i_2=1,\ldots,N). \tag{5}$$

Based on Eqs. (2)–(4), matrices R_{i_2}'s can be recursively computed using

$$R_N=-B_{N-1}(A_N^{(1)})^{-1}, \tag{6}$$

$$R_{i_2}=-B_{i_2-1}[A_{i_2}^{(1)}+R_{i_2+1}C_{i_2+1}]^{-1},\quad (i_2=N-1,\ldots,1 \text{ and } \mathrm{mod}(i_2)\ne0). \tag{7}$$

In case of $\mathrm{mod}(i_2)=0$, Eq. (2) can be rewritten as follows:

$$\pi_{i_2-1}B_{i_2-1}+\pi_{i_2}A_{i_2}^{(1)}+\pi_{i_2+1}C'_{i_2+1}=0. \tag{8}$$

with

$$C'_{i_2+1}=C_{i_2+1}+C_{i_2+1,*}+\sum_{l=2}^{\aleph}\prod_{j=2}^{l}R_{i_2+j}C_{i_2+l,*}, \tag{9}$$

as

$$\pi_{i_2+l}=\pi_{i_2+1}R_{i_2+2}\ldots R_{i_2+l-1}R_{i_2+l}=\pi_{i_2+1}\prod_{j=2}^{l}R_{i_2+j},$$

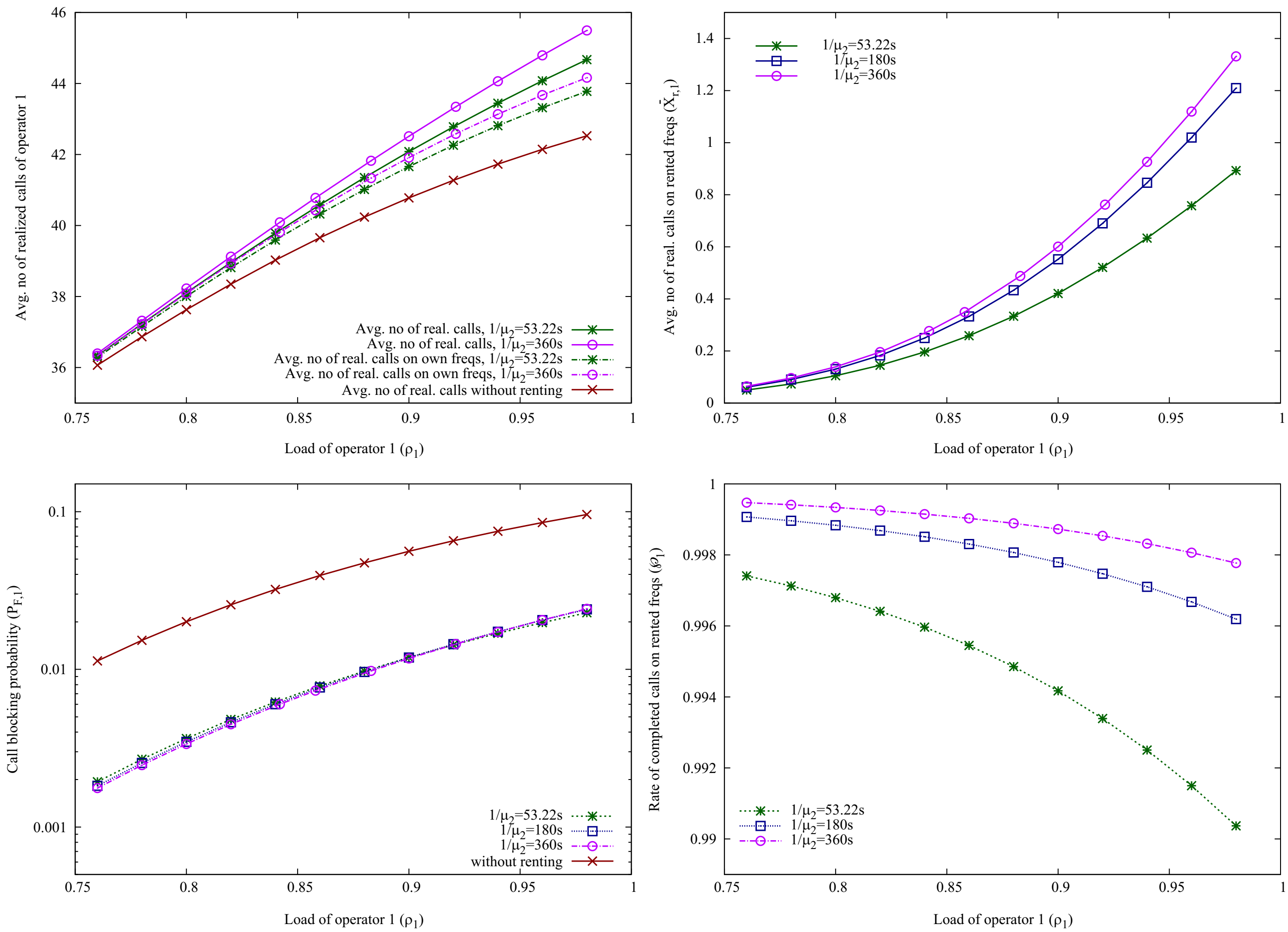

Fig. 4. Performance measures for $n_1 = n_2 = 6$, $1/\mu_1 = 180$ s and $\rho_2 = 0.7$.

As a consequence, we obtain

$$R_{i_2} = -B_{i_2-1}[A^{(1)}_{i_2} + R_{i_2+1}C'_{i_2+1}]^{-1}, \quad (N_2 \le i_2 < N \text{ and } \mathrm{mod}(i_2) = 0). \tag{10}$$

Eq. (1) can be rewritten as

$$\pi_0(A^{(1)}_0 + R_1 C_1) = 0. \tag{11}$$

To compute π_0 we utilize the normalization equation $\sum_{i_2=0}^{N} \pi_{i_2}\mathbf{e}_{i_2} = 1$ and Eq. (11), where $\mathbf{e}_{i_2}$ is the vector of size i_2 with all elements being equal to one. Next, substituting π_0 into (5) we get the stationary probabilities.

Remark. The computation complexity of solving a linear equation constructed from Eq. (11) and the normalization equation is $O((N+1)^3)$. The complexity of computing of R_{i_2}, $i_2 = 1, \ldots, N$, is $O((N+1-\lceil i_2/\aleph\rceil\aleph)^3)$ because the inversion of matrices is a dominant factor in the computation. The computation complexity regarding Eq. (5) is only $O(\sum_{i_2=0}^{N-1}(N+1-\lceil i_2/\aleph\rceil\aleph)(N+1-\lceil i_2+1/\aleph\rceil\aleph))$. Therefore, the computational complexity for the steady state probabilities is $O(\sum_{i_2=0}^{N}(N+1-\lceil i_2/\aleph\rceil\aleph)^3 + \sum_{i_2=0}^{N-1}(N+1-\lceil i_2/\aleph\rceil\aleph)(N+1-\lceil i_2+1/\aleph\rceil\aleph))$.

4.2. Performance measures

The purpose of the performance evaluation in this paper is to compute some important performance measures characterizing the operation of the spectrum renting scheme based on the modelling assumptions (see Section 3). In this paper, we deal with performance measures that can be derived from the stationary distribution.

The call blocking probability is often used to characterize the quality of service of mobile cellular networks (it is called a grade of service provisioned to subscribers). When a handover call arrives to operator 1, it is blocked if there is no any available channel in own and rented frequency bands and there is no free frequency band at operator 2. Using the PASTA (Poisson Arrivals See Time Averages) rule (Kleinrock, 1975), the blocking probability of handover calls at operator 1, defined as $P_{H,1}$, can be expressed as

$$P_{H,1} = \sum_{i_2=0}^{N_2} p_{f(i_2),i_2}.$$

Similarly, the blocking probability of handover calls at operator 2 can be given by

$$P_{H,2} = \sum_{i_1=0}^{N_1} p_{i_1,f(i_1)}.$$

A new call at operator 1 is blocked if there is no vacant channel for this new call due to the applied FGC policy and operator 1 cannot borrow a new free frequency from operator 2. Therefore the blocking probability of fresh calls at operator 1, defined as

$P_{F,1}$, can be expressed as

$$P_{F,1} = \sum_{i_2=0}^{N} \sum_{i_1=0}^{f(i_2)} p_{i_1,i_2}(1-\beta_{1,i_1}),$$

and similarly, the blocking probability of fresh calls at operator 2 can be obtained:

$$P_{F,2} = \sum_{i_1=0}^{N} \sum_{i_2=0}^{f(i_1)} p_{i_1,i_2}(1-\beta_{2,i_2}).$$

From the viewpoint of operators, it is important to compute quantities characterizing the utilization of frequency bands. Now let us consider the average number of realized calls, defined as $\overline{X_k}$ at operator k, $(k=1,2)$. This measure is defined as the average number of accepted fresh calls and handover calls at operator k. We have

$$\overline{X_1} = \sum_{i_2=0}^{N} \sum_{i_1=0}^{f(i_2)} p_{i_1,i_2} * i_1,$$

and

$$\overline{X_2} = \sum_{i=0}^{N} \sum_{i_2=0}^{f(i_1)} p_{i_1,i_2} * i_2.$$

The average number of calls on rented frequency bands at each operator, $\overline{X_{r,k}}$, $(k=1,2)$, is defined as

$$\overline{X_{r,1}} = \sum_{i_2=0}^{N_2-\aleph} \sum_{i_1=N_1+1}^{f(i_2)} p_{i_1,i_2}(i_1-N_1),$$

and

$$\overline{X_{r,2}} = \sum_{i_1=0}^{N_1-\aleph} \sum_{i_2=N_2+1}^{f(i_1)} p_{i_1,i_2}(i_2-N_2).$$

The average number of calls realized in own frequency bands by each operator, $\overline{X_{o,k}}$, $(k=1,2)$, is defined as

$$\overline{X_{o,1}} = \overline{X_1} - \overline{X_{r,1}} = \sum_{i_2=0}^{N} \sum_{i_1=0}^{f(i_2)} p_{i_1,i_2} * \min(i_1, N_1),$$

and

$$\overline{X_{o,2}} = \overline{X_2} - \overline{X_{r,2}} = \sum_{i_1=0}^{N} \sum_{i_2=0}^{f(i_1)} p_{i_1,i_2} * \min(i_2, N_2).$$

Calls located on rented frequency bands may be forced to leave. The rate of successfully completed calls on rented frequency bands, $\wp_1$ and $\wp_2$, can be used to characterize the "good luck" of the opportunistic renting scheme as well. We have

$$\wp_1 = 1 - \frac{\text{Total forced termination rate of operator 1}}{\text{Total connection rate of operator 1 on rented frequency bands}}$$

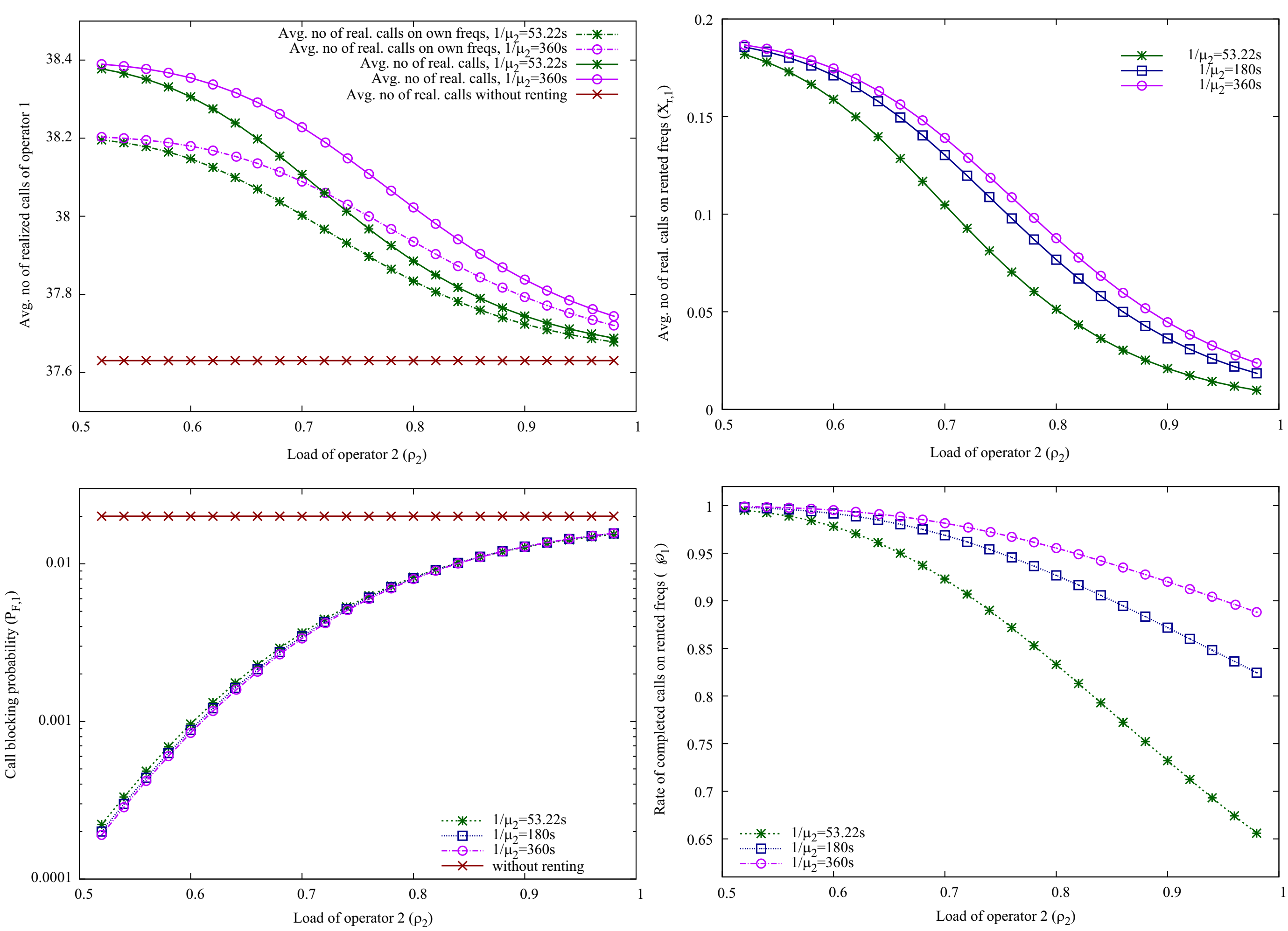

Fig. 5. Performance measures for $n_1 = n_2 = 6$, $1/\mu_1 = 180$ s and $\rho_1 = 0.8$.

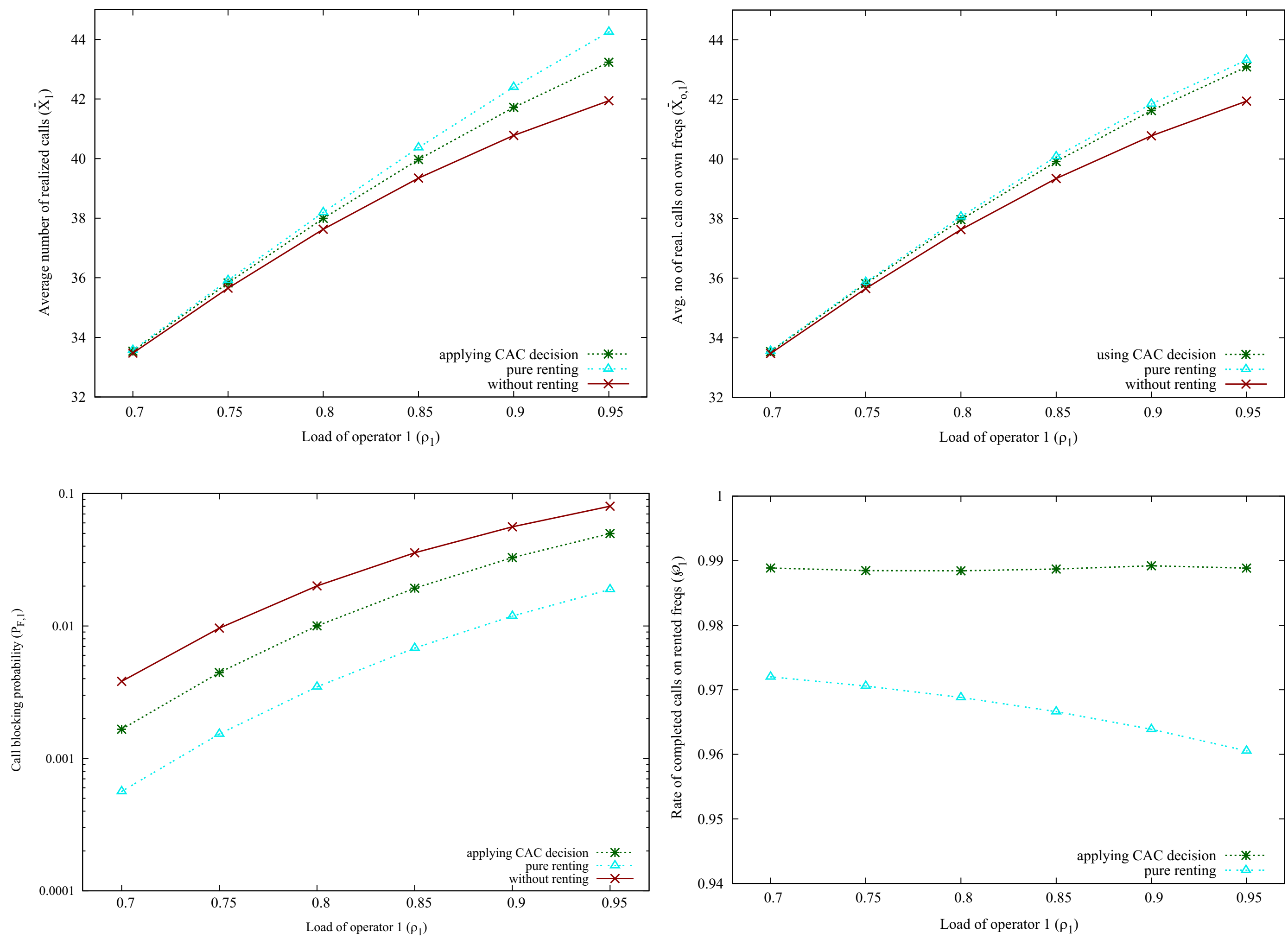

Fig. 6. Performance measures for $n_1 = n_2 = 6$, $1/\mu_1 = 1/\mu_2 = 53.22$ s, $\rho_2 = 0.7$.

$$= 1-\frac{\sum_{l=0}^{n_2-1}\sum_{j=0}^{\aleph-1}(\aleph-j)\lambda_{2,\aleph l}p_{f(\aleph l)-j,\aleph l}}{\sum_{i_2=0}^{N_2-\aleph}\sum_{i_1=N_1}^{f(i_2)-1}\lambda_{1,i_1}p_{i_1,i_2}}.$$

Similarly, we obtain

$$\wp_2 = 1-\frac{\sum_{l=0}^{n_1-1}\sum_{j=0}^{\aleph-1}(\aleph-j)\lambda_{1,\aleph l}p_{\aleph l,f(\aleph l)-j}}{\sum_{i_1=0}^{N_1-\aleph}\sum_{i_2=N_2}^{f(i_1)-1}\lambda_{2,i_2}p_{i_1,i_2}}.$$

5. Numerical results for a case with two operators

In practice, each area is dimensioned such that subscribers are served with a minimum possibility of violating the grade of service. As a rule of thumb, the blocking probability of fresh calls (the grade of service) should be around 1% in a specific area and the blocking probability of handover calls should be less than the blocking probability of fresh calls. In this section, we investigate a situation when the increase of offered traffic leads to the degradation of the grade of service without the rent of spectrum and show that the opportunistic spectrum renting scheme can be an efficient tool to keep the target grade of service.

5.1. A simulation model with lognormal distributed holding times

The analysis of call traces in mobile cellular networks, performed by Jedrzycki and Leung (1996), showed that the holding times of calls in cellular networks have the lognormal distribution. Therefore, we compare results obtained by our analytical model and a simulation model where the channel holding times follow the lognormal distribution with the mean of 3.29 s and the standard deviation of 1.17 s in the simulation runs. Note that parameter values are taken from Jedrzycki and Leung (1996). Simulation runs were performed with the confidence level of 99.9%. The confidence interval is $\pm 0.6\%$ of the collected data. In Fig. 3 we plot performance measures vs loads $\rho_1 = \lambda_1/(N_1\mu_1)$ and $\rho_2 = \lambda_2/(N_2\mu_2)$ for $n_1 = 6$, $n_2 = 6$ and $\aleph = 8$. From the curves, we observe the excellent agreement between analytical and simulation results, which shows that our model with the exponential distribution of channel holding times can be used to predict the performance of a cell where an opportunistic spectrum pooling is applied and the holding times of calls follow the lognormal distribution.

5.2. Impact of opportunistic spectrum renting

Figures 4 and 5 clearly show the advantage of the opportunistic spectrum pooling scheme. The blocking probability is decreased by one order of magnitude in a large range of the offered traffic. For example, Fig. 4 shows that the blocking probability decreased from 2% to 0.36% for $\rho_1 = 0.8$, $\rho_2 = 0.8$ and $1/\mu_1 = 180$ s when spectrum renting is applied. It is worth emphasizing that *the utilization of own frequency bands* is increased due to spectrum renting (i.e., the average number of calls realized on own frequency bands is increased in comparison to the average number of established calls when no renting is performed). For example, at $\rho_1 = 0.95$ and $1/\mu_2 = 360$ s, the average number of calls established on own frequency is approximate 43.41 and 41.94 for the renting case and the no renting case, respectively (see Fig. 4). The increase in the average number of calls realized on own frequency bands is more than the average number of calls allocated on rented frequency bands (at $\rho_1 = 0.95$ and $1/\mu_2 = 360$ s, the increase is $43.41 - 41.94 = 1.47$, while the average number of calls on rented frequency bands is 1.02).

On the one hand, the positive impact of a spectrum renting on the blocking probability and the average number of calls realized on own frequency bands is indeed a good news for operators. For instance, the average number of calls increased from 37.63 to 38.31 for $\rho_1 = 0.8$, $\rho_2 = 0.6$ and $1/\mu_2 = 53.22$ s applying spectrum renting (see Fig. 5).

On the other hand, the price of opportunistic spectrum renting is that calls may be forced to leave a rented frequency and only a portion of subscribers who are allocated in rented frequency bands can successfully complete their calls (see Figs. 4 and 5). Therefore, a procedure is needed to balance the annoyance concerning the interrupted calls when the renter takes back an opportunistically used frequency band, which will be presented in Section 5.3.

5.3. Balancing the forced blocking probability and the blocking probability of calls

In what follows, to ease the comprehension we present a procedure from the aspect of operator 1 which acts as a tenant. To increase the number of calls admitted, the opportunistic renting of free bands from operator 2 is performed by operator 1. Although operator 1 reallocates ongoing calls realized in a rented band to its own frequency band when a call allocated in an own band departs, it is still possible that ongoing calls are forced to leave the system because operator 2 takes back a frequency band. This is the price of opportunistic spectrum renting indeed, which may cause an inconvenience for subscribers. To minimize the number of the interrupted calls it is reasonable to control the number of calls that are placed in rented frequency bands on the one hand, but the advantage of renting frequency bands should be preserved on the other hand. That is, operator 1 should not allocate calls in a specific rented frequency band if it can “foresee” that the specific band will be taken back by operator 2 soon. Therefore, the essential step of a proposed call admission control procedure is to continuously monitor the tendency regarding the number of free frequency bands owned by operator 2. Operator 1 computes the exponential weighted moving average $\nu_2(t)$ of the number of free bands $B_2(t)$ that belong to operator 2 and are not used by any operator at time instant t_m is

$$\nu_2(t_m) \leftarrow (1 - 2/(w_2+1))\nu_2(t_{m-1}) + 2B_2(t_m)/(w_2+1), \quad (12)$$

where w_2 is the window size (weight) chosen by operator 1. Note that w_2 reflects the number of the past changes is closely considered in Eq. (12).

The following call admission control (CAC) decision is proposed:

- if $I_1(t) < N_1$ a call is admitted, where $I_1(t)$ is the number of busy channels handled by operator 1,
- if $I_1(t) \geq N_1$
  - if$(B_2(t) > 1)$ or $((B_2(t) = 1)$ and $(I_1(t) \bmod (n) \neq 0))$, an arriving call is admitted,
  - else if $0 \leq \nu_2 \leq 1$, rejecting an arriving call with probability $1 - \max_p \times \nu_2$.

To predict whether the specific band will be taken back by operator 2, operator 1 should incorporate a tendency regarding the change of the number of free bands owned by operator 2 (observed by operator 1). This can be done by the appropriate choice of sampling points t_m. It is normally expected that $\nu_2(t)$ is updated at every instant when $B_2(t)$ is changed. Since a CAC decision is made by operator 1 at the arrival instants of calls, it is reasonable to choose the arrival instants of calls at operator 1 as sampling instants as well.

The impact of the CAC decision is illustrated in Fig. 6 where results are obtained through simulations. In the simulations, the confidence level is 99.9%, the confidence interval is $\pm 0.6\%$ of the collected data, while parameter $\max_p$ is 0.9 when the CAC is applied. The CAC limits the allocation of incoming calls in rented frequency bands in order to reduce a chance that ongoing calls are forced to leave the system.

From Fig. 6, it is observed that limiting the number of calls allocated in rented frequency bands has a small impact on the utilization of own frequency bands. The ratio of successfully completed calls increased from 96.88% to 98.32% for $\rho_1 = 0.8$ when the CAC is used. This result means that the forced termination probability decreased by 46% in case of calls served on rented frequency bands. Note that the price of the increased protection for the ongoing calls is the increased blocking probability of fresh calls. Similarly for $\rho_1 = 0.8$, the blocking probability increased from

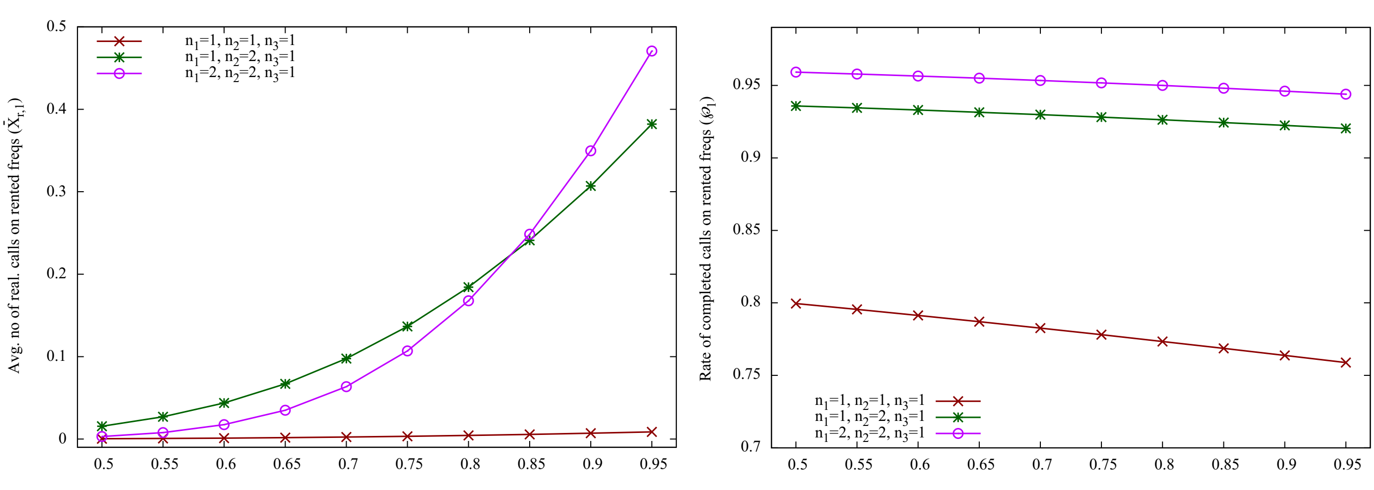

Fig. 7. Performance measures for $\rho_2 = 0.5$, $\rho_3 = 0.7$, $\aleph = 8$ and $\mu_1 = \mu_2 = \mu_3$.

0.35% to 0.83%, when the CAC decision is applied. The investigation demonstrates that operators can further fine-tune the operation of the opportunistic spectrum renting scheme to balance between the quality of service and the annoyance experienced by users.

6. A case with the number of providers more than two

If a specific operator wants to use frequency bands from other operators in a case of $K > 2$, it is reasonable that each operator should specify a rule regarding a preferable tenant operator (e.g., in the form of a preference list) or a tenant operator is randomly selected. Furthermore, a rule to claim back a frequency band is also needed when several frequency bands are rented by multiple operators. Based on the rules the state transitions balance equations can be determined, then the steady state distribution and the performance measures can be computed with the help of standard techniques (Takacs, 1962; Bolch et al., 2006) or a method presented in Section 4.

From numerical results presented in Section 5, it is observed that the increase of offered traffic to some extent can be handled by renting one frequency band for a specific operator in the most of cases to relieve the temporary capacity shortages. Therefore, it is suggested that

(i) each operator allocates one dedicated frequency band. These dedicated frequency bands form a pool for opportunistic spectrum renting. Each operator has a right to use one frequency band from the pool at any time and opportunistically uses other bands from the pool if these bands are not occupied by other operators. This approach will further simplify the operation of opportunistic spectrum renting because an effort to maintain the state of frequency bands is less than the general case. Furthermore, a computational complexity is reduced because the number of states in a Markov chain is decreased;

(ii) alternatively operators cooperate in a “peer-to-peer way”, which is the case presented in Section 4.

We present the analysis for a case when three operators apply an opportunistic spectrum renting with a common pool (alternative (i)) in the e-companion of this paper. In Fig. 7, we plot the rate of completed calls on rented frequency bands ($\wp_1$) and the number of realized calls on rented frequency bands ($\overline{X}_{r,1}$) of operator 1 vs the load offered to operator 1, $\rho_1 = \lambda_1/(N_1\mu_1)$, for $\rho_2 = \lambda_2/(N_2\mu_2) = 0.5$, $\rho_3 = \lambda_3/(N_3\mu_3) = 0.7$, $\mu_1 = \mu_2 = \mu_3$ and $\aleph = 8$. Again, Fig. 7 confirms our observation from the numerical study in Section 5, which leads us to the proposal (alternative (i)) of a common pool with dedicated frequency bands.

7. Conclusions

We have proposed the spectrum pooling cooperation scheme for mobile cellular service providers operating in a specific area. We have presented the analytical framework to evaluate the performance of the opportunistic spectrum pooling scenario between operators. From our intensive numerical investigation, we have revealed an observation that the increase of offered traffic to some extent can be handled by renting one frequency band for a specific operator to relieve the temporary capacity shortages. We have proposed the flexible and simple cooperation scheme when more than two operators are involved. The viability of our proposal is also confirmed by a numerical study. The proposed scheme along with its simplicity can provide a good opportunity for operators to enhance a grade of service provisioned to subscribers, which could raise an incentive for operators to cooperate together.

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Appendix A. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.jnca.2015.02.007.

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