

# A Finite-Source Queuing Model for Spectrum Renting in Mobile Cellular Networks

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**Abstract**—Nowadays, spectrum renting is a frequently investigated research topic with the aim of improving the efficiency of spectrum usage in mobile cellular networks.

In this paper, we propose a finite-source queuing model, in which service providers may rent each other's unutilized frequency bands. Our model contains two source pools: one for subscribers generating fresh calls in the investigated mobile cell, and another for modeling handover calls. The model considers the traveling of subscribers over the borders of the investigated mobile cell, too. Moreover, we take into account the retrial phenomenon caused by the impatience of subscribers.

**Index Terms**—spectrum renting, finite-source, retrial queues, handover, impatience;

## I. INTRODUCTION

The basic idea of spectrum renting was first proposed by Mitola [1], to aid the efficient usage of the available radio spectrum and alleviate the effects of its scarcity. Since the Federal Communications Commission's Spectrum Policy Task Force [2] proposed the dynamic/liberalized access to the spectrum in 2002, many researchers have dealt with the realization of spectrum renting [3], [4], [5], [6], [7]. Besides, some queuing models for spectrum renting were also proposed. In [8] and [9], the authors assume that the speech channels can be rented in one unit, which is not a valid assumption, since each block of speech channels should be controlled by a single network operator [10]. Do et al. considered first this technology aspect in their infinite-source model [10], but they did not consider the retrial phenomenon in their model. In [11], an opportunistic spectrum access model was proposed, which incorporated the presence of two competing service providers. However, the handover process and the impatience of the users were not touched in that paper.

The retrial queues have been widely studied, too, for the evaluation of the resource contention problem in mobile cellular networks. The works in this area can be classified based on the number of traffic sources. Infinite-source models were presented, e.g., in [12], [13], [14], [15], [16], while finite-source works can be found, e.g., in [17], [18], [19], [20], [21]. The assumption on the infinite number of sources makes the evaluation of the model easier. However, the fact that the number of subscribers in a specific mobile cell is finite, justifies the construction of finite-source models. In [22], the authors proposed the use of the retrial queues to model

spectrum renting. The model considers the finite size of the subscribers' population, their impatience, and has a queue for the subscribers who requested the outbound service. Moreover, the authors took the renting fee into account, too, for the optimization of the renting process. As it was mentioned in that paper, several model variants could be derived from [22] to investigate additional aspects of spectrum renting.

Based on [22], we set up a finite-source queuing model with two source pools. The additional source pool allows us to investigate also the handover calls, which were not mentioned in [22]. Moreover, our model considers the traveling of subscribers by modeling the transitions over the borders of the investigated mobile cell.

The rest of the paper is organized as follows. Section II introduces the key aspects of our model. Section III describes our system model, while numerical results are presented in Section IV. Finally, Section V concludes the paper.

## II. BACKGROUND

In this section, we give a short overview of the key aspects which we took into account in our model: the organization of speech channels, the spectrum renting, the impatience of subscribers and the handover process.

- In mobile cellular networks, service providers get exclusive access to certain frequency bands on spectrum auctions held by the government. Each service provider divides its coverage area to regular shaped cells, and assigns a number of frequency bands to each cell. Using different multiple access techniques, such as the Frequency and the Time Division Multiple Access (FDMA, TDMA), and the Code Division Multiple Access (CDMA), each frequency band hosts regularly 8 full-rate or 16 half-rate speech channels [23], [24]. Therefore, a finite number of speech channels is available in each cell.
- Since the right to frequency usage is expensive, improving the utilization in mobile cellular networks is a critical task. Spectrum renting can be used for alleviating the negative effects of temporary capacity shortages [5], [10], [11]. With spectrum renting, service providers may rent each other's unutilized frequency bands to reduce the blocking probability of calls.

- As it was mentioned in the literature [10], [18], [25], behavioral psychology of using the mobile cellular network services includes repeated attempts and abandonments. When a call is not admitted due to all channels are occupied, the subscriber initiates a repeated attempt later. The abandonment means that after a certain time, the subscribers gives up without being served.
- When the signal strength of a base station which a traveling user is connected to, decreases below a certain value, another base station with appropriate signal strength must be chosen for serving the ongoing call. This process is termed as handover. Since the break of the connection is annoying for traveling subscribers, a protection procedure must be applied for handover calls (e.g., the well-known guard channel policy [26]).

### III. SYSTEM MODEL

Fig. 1 illustrates our system model. The model incorporates the handover process with fractional guard channel (FGC) policy, an orbit for the impatient users, and the blocks of channels, considering the own and the rented blocks, too. Moreover, the model contains two finite sources *i*) for generating the fresh calls within the bounds of the investigated mobile cell, and *ii*) for the handover calls.

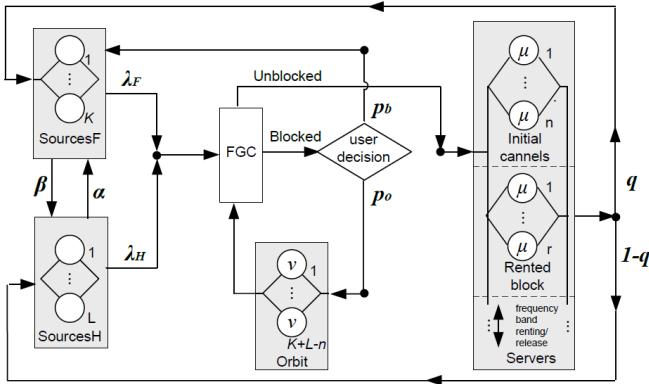


Fig. 1: The system model of the retrial queue

#### A. Model Operation

From the mobile cell's point of view, a finite number of  $K$  subscribers belongs to one mobile cell, generating fresh calls. Being in the *SourcesF* state, each subscriber initiates a fresh call with rate  $\lambda_F$ .

Similarly, a finite number of  $L$  subscribers belongs to the investigated mobile cell, being physically out of the cell, generating handover calls. So, the handover calls are generated from state *SourcesH* with rate  $\lambda_H$  on the average by each subscriber.

In order to protect the handover calls, a FGC policy is applied for the incoming calls. If a call is admitted, a channel is immediately allocated for it, either in an own frequency band, or in a rented one (state *Servers* in Fig. 1). The call duration is exponentially distributed with parameter  $\mu$ , after

which the channel is released and the subscriber returns to the *Sources* state.

If the call is not admitted, the subscriber has two options: *i*) he/she joins the orbit (state *Orbit* in Fig. 1) with probability  $p_o$ , and retries to get a free channel with retrial rate  $v$ ; or *ii*) he/she gives up and becomes idle with probability  $p_b = 1 - p_o$ .

#### B. Decision about Renting and Releasing a Frequency Band

Do et al. proposed two thresholds for the number of free channels in [10] to determine when to rent and release frequency bands, and we apply their approach in our model.

When the number of free channels decreases to a certain  $t_1$  value, the network operator initiates the frequency renting procedure. The frequency renting is successful in our model with probability  $p_r$ , while it fails with probability  $p_f = 1 - p_r$ . In the latter case, the network operator retries to rent a frequency band with rate  $v_r$  as long as the number of free channels does not exceeds  $t_1$ .

On the other hand, when the number of free channels increases to  $t_2 + r$ , then the block of  $r$  channels are given back to the owner after an exponentially distributed release time with mean  $\mu_r$ .

To find the appropriate parameter setting, we have to take into account the blocking probability of fresh calls and handover calls, and the renting fee. To facilitate this decision, we take into account the average profit rate, too, in our investigations.

#### C. Fractional Guard Channel Policies

In order to minimize the interruption of ongoing calls, network operators exclusively reserve some channels for handover calls [26]. A generalization of this concept is the FGC policy proposed in [27]. To ease the further negotiation, we collected the notations in Table I.

TABLE I: Notations

Parameter	Symbol
$I(t)$	The number of occupied channels in a specific cell.
$J(t)$	The number of rented frequency bands.
$\beta_{i,j}$	The probability of accepting a call for $I(t) = i$ and $J(t) = j$ .
$n$	The number of own channels.
$n_R$	The number of rented frequency bands.
$N_j$	The number of available channels. $(N_j = n + jn_R \text{ for } J(t) = j)$
$g$	The real number of guard channels.

Note that in case of a handover call,  $\beta_{i,j} = 1$ . Based on [28], several FGC policy variants can be distinguished. Since Do et al. showed in [10] that the Limited average FGC (LFGC) is the most convenient for protecting handover calls, we applied this policy and the Non Prioritization Scheme (NPS) in this paper. These policies can be defined as follows [29]:

- Limited average FGC (LFGC)*, where  

$$\beta_{i,j} = \begin{cases} 1 & \text{if } 0 \leq i \leq N_j - \lfloor g \rfloor - 2, \\ 1 - g + \lfloor g \rfloor & \text{if } i = N_j - \lfloor g \rfloor - 2, \\ 0 & \text{else.} \end{cases}$$
- Non Prioritization Scheme (NPS)*, where  $\beta_{i,j} = 1$ .

#### IV. NUMERICAL RESULTS

The state space of the Continuous-Time Markovian Chain (CTMC) describing our model is finite, so the steady state probabilities surely exist. Taking into account, that the state space of this CTMC is very large, it is difficult to determine the performance indices of the system in a traditional way of solving the steady-state equations. Therefore, we use the *MOSEL-2* software tool to solve the underlying steady state equations of the CTMC. The *MOSEL-2* tool uses a high-level modeling language that provides a very convenient way for system description [30]. It focuses on the formal system description and exploits the power of various existing and well-tested packages (*MOSES* [31], *SPNP* [32], [33] and *TimeNET* [34]) for numerical evaluation.

In Table II, we collected the parameter settings we used in our investigations.

TABLE II: The applied parameter setting

Parameter	Symbol	Value
Normalized traffic intensity	$\rho_0 = \frac{K\lambda}{n\mu}$	[0.1 .. 0.9]
Call generation rate	$\lambda$	
Fresh call generation rate	$\lambda_F$	$\frac{2}{3}\lambda$
Handover call generation rate	$\lambda_H$	$\frac{1}{3}\lambda$
Initial no. of subscribers in SourcesF	$K$	100
Initial no. of subscribers in SourcesH	$L$	50
Real number of reserved channels	$g$	2.9
Retrial rate	$\nu$	1
Prob. that subscriber gives up	$p_b$	0.1
Impatience rate	$\eta$	1/300
Prob. that imp. user goes back to orbit	$p_{Io}$	0.8
No. of channels per block	$r$	8
Number of channels without renting	$n$	$2 * r$
Service rate	$\mu$	1/53.22
Block renting threshold	$t_1$	3
Block renting rate	$\lambda_r$	1/5
Block release threshold	$t_2$	6
Block release rate	$\mu_r$	1
Prob. for successful renting	$p_r$	0.8
Block rental retrial rate	$\nu_r$	1/7

Do et al. showed in [10] that using exponential distribution is a good approximation for modeling the channel holding times, which follow the log-normal distribution [35]. To get a mathematically tractable model, we also used the exponential distribution for channel holding times with the mean of 53.22s, which was determined observing real data traffic [35].

In the following figures, we depict the results applying LFGC policy for protecting the handover calls, and the case when no protection was applied (NPS). Moreover, Figures 2-5 show the results besides the use of spectrum renting, and the no renting case, when  $p_r = 0$ . The results clearly show that the possibility of spectrum renting significantly decreases the blocking probabilities. Accordingly, the mean orbit size also decreases applying spectrum renting. Finally, the use of spectrum renting increases the average profit rate (APR), which is an evident financial advantage.

In the following, we focus on the results concerning the handover calls' protection, since touching this aspect is a remarkable novelty in our contribution.

Figures 2 and 3 show the blocking probabilities of fresh and handover calls, respectively, as a function of the normalized traffic intensity. We can observe that applying LFGC policy, we obtain an increased protection for handover calls, while the price of this protection is the increased blocking probability of fresh calls compared to the case when NPS is used.

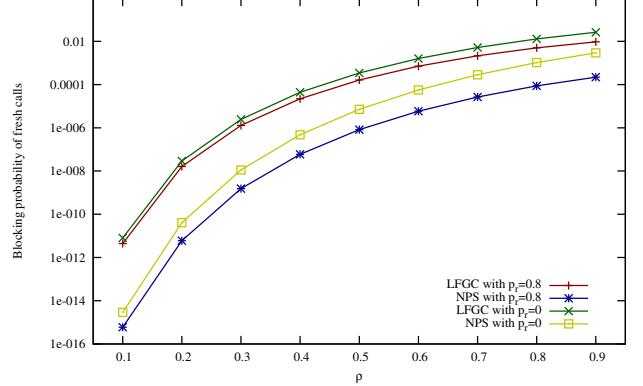


Fig. 2: Blocking probability of fresh calls as a function of the normalized traffic intensity

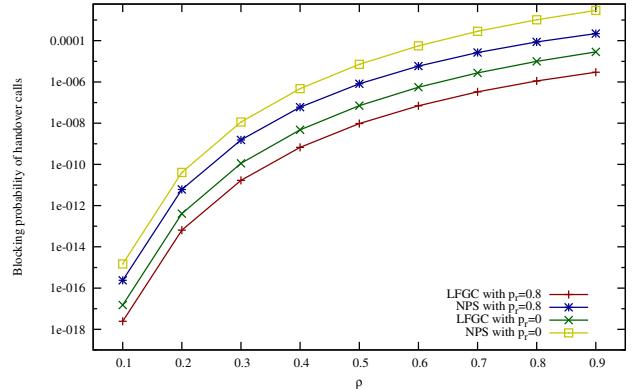


Fig. 3: Blocking probability of handover calls as a function of the normalized traffic intensity

In Figure 4, the mean orbit size is shown. Since the subscribers rapidly leave this state (either due to retrial or giving up), the number of subscribers is relatively low in the orbit, even if the traffic increases. Due to the increased blocking probability caused by protecting handover calls, the mean orbit size is higher when LFGC policy was applied than in case of NPS.

To measure the financial effect of the handover calls' protection, we illustrated the APR as a function of the normalized traffic in Fig. 5. Besides the case of NPS, we depicted two graphs with different values of LFGC parameter  $g$ . We can realize that protecting the handover calls has little impact on APR, when number of guard channels is 2.9. Using  $g = 6.3$ , the difference between the investigated policies becomes significant as the normalized load increases (the protection of handover calls with relatively high number of guard channels

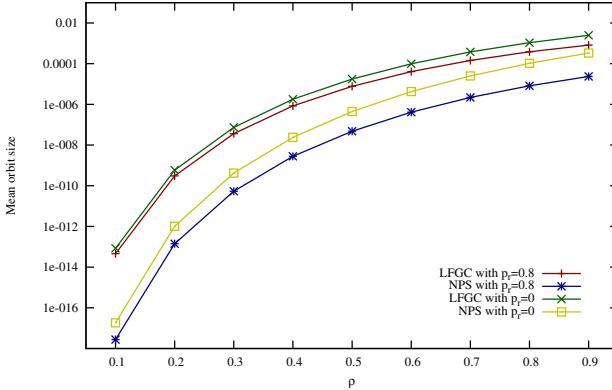


Fig. 4: The mean orbit size as a function of the normalized traffic intensity

must be paid). However, the operators are able to fine-tune the value of  $g$  in view of the expected profit, too.

For the more precise investigation of LFGC parameter  $g$ , we investigated the above mentioned performance indices as a function of  $g$  (see Figures 6-9). To ease the comparison with the case of NPS, we also represent the constant values of NPS policy in Figures 6-9. As we can see in Fig. 6 and 7, the operators should find a trade-off between the blocking probabilities of fresh calls and handover calls, while the effect of the value of  $g$  on the APR is not significant in this scenario until  $g$  increases to around 4 (Fig. 9).

## V. CONCLUSIONS

In this paper, we described a queuing model with finite number of subscribers for modeling spectrum renting. The results showed that applying spectrum renting, the blocking probabilities decrease, while the increasing value of APR justifies the financial benefits of spectrum renting.

To the best of our knowledge, our model is the first finite-source queuing model for modeling spectrum renting in wireless cellular networks that incorporates the handling of handover calls' protection, together with the modeling subscribers' mobility and the retrial phenomenon caused by the impatience of subscribers. Our investigations showed that with appropriate parameter setting, the mobile operators can make a trade-off between the blocking probability of fresh calls and handover calls, while the average profit rate can be preserved by controlling the number of guard channels.

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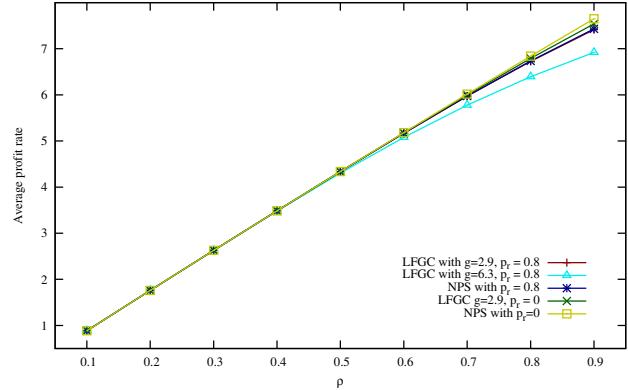


Fig. 5: The average profit rate as a function of the normalized traffic intensity

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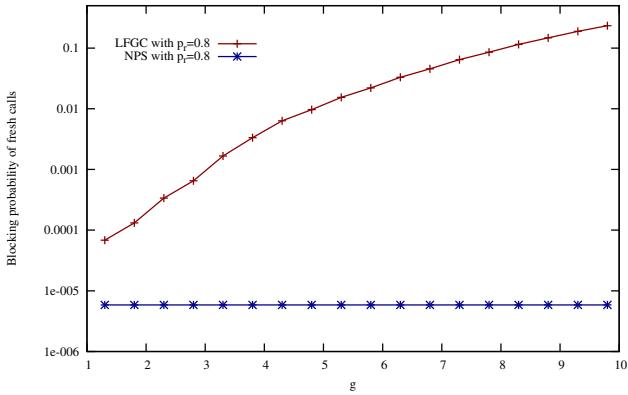


Fig. 6: The blocking probability of fresh calls as a function of the number of guard channels

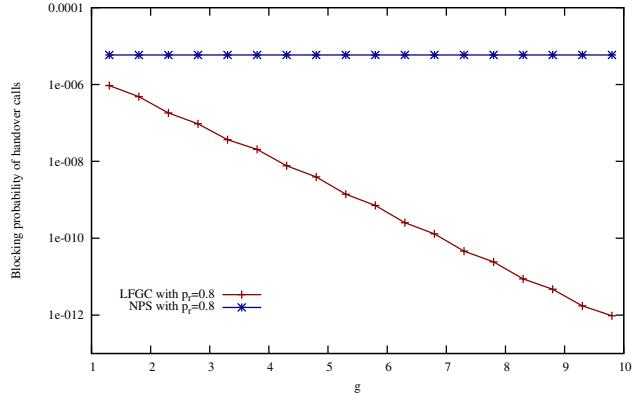


Fig. 7: The blocking probability of handover calls as a function of the number of guard channels

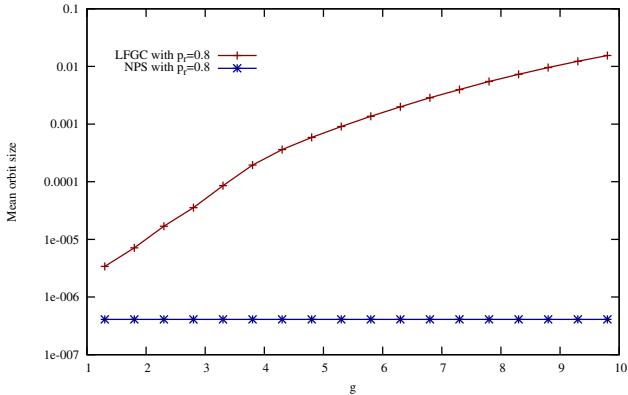


Fig. 8: The mean orbit size as a function of the number of guard channels

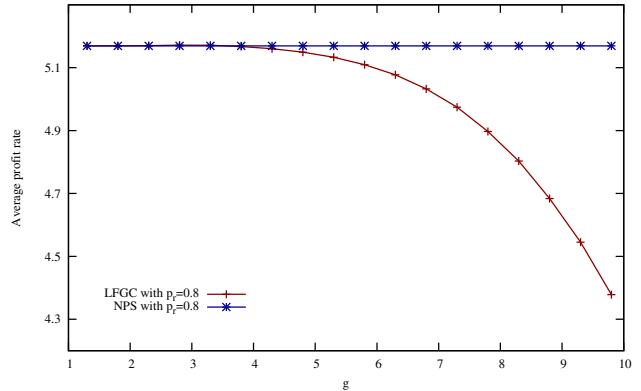


Fig. 9: The average profit rate as a function of the number of guard channels

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